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Axisymmetric Buckling of Antisymmetrically Laminated Spherical Caps

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Introduction

LAMINATED shells are finding increased applications in the aerospace, automobile, and power industries. These shells may be subjected to severe operational conditions, causing large deformations. There are not many investigations reported on the analysis of laminated spherical caps undergoing large axisymmetric deformations. Recently Xu[1] has investigated, using shallow shell theory, the large deformation problem of symmetrically laminated shallow spherical shells using the Bessel-Fourier series approach. The present investigation is concerned with the analysis, using deep shell theory, of large axisymmetric deformation behavior of antisymmetrically laminated cross-ply spherical shells. Estimates of snap pressures for symmetrically laminated caps using deep shell theory are compared with those of Xu.[1] Some new results with respect to antisymmetrically laminated caps are also presented.

Mathematical Formulation

Assuming the polar orthotropic cross-ply spherical cap to be undergoing moderately large axisymmetric deformations, the nonlinear strains and curvatures of the reference surface are given by

$$e_s^0 = \frac{du}{ds} + \frac{w}{R} + \frac{1}{2}\left\{\frac{dw}{ds} - \frac{u}{R}\right\}^2$$

$$e_\theta^0 = \frac{\cot\phi}{R}u + \frac{w}{R}$$

$$e_{sz}^0 = \left\{\frac{dw}{ds} + \alpha - \frac{u}{R}\right\}$$

$$\lambda_s^0 = \frac{d\alpha}{ds} - \frac{\alpha}{R}\left\{\frac{dw}{ds} - \frac{u}{RT}\right\} + \frac{1}{2}\frac{1}{R}\left\{\frac{dw}{ds} - \frac{u}{R}\right\}^2$$

$$\lambda_\theta^0 = \cot\phi\,\frac{\alpha}{R} \tag{1}$$

where u and w are displacements in the meridional and radial directions, respectively, and α the rotation of the normal (Fig. 1). R is the radius of the shell.

The governing equilibrium equations for axisymmetric deformations of laminated spherical shells are derived using virtual work principles and are given by

$$\frac{dN_s}{ds} + (N_s - N_\theta)\frac{\cot\phi}{R} + \frac{Q_s}{R} = \frac{N_s}{R}\left\{\frac{u}{R} - \frac{dw}{ds}\right\} - \frac{M_s}{R^2}\left\{\frac{u}{R} - \frac{dw}{ds} - \alpha\right\}$$

$$\frac{dM_s}{ds} + (M_s - M_\theta)\frac{\cot\phi}{R} - Q_s = \frac{M_s}{R}\left\{\frac{u}{R} - \frac{dw}{ds}\right\}$$

$$\begin{aligned}\frac{N_s + N_\theta}{R} - \frac{dQ_s}{ds} - Q_s\frac{\cot\phi}{R} = q &- \frac{d}{ds}\left\{\left(N_s - \frac{M_s}{R}\right)\left(\frac{u}{R} - \frac{dw}{ds}\right)\right\} \\ &- \frac{\cot\phi}{R}\left\{\left(N_s - \frac{M_s}{R}\right)\left(\frac{u}{R} - \frac{dw}{ds}\right)\right\} \\ &- \frac{d}{ds}\left(\frac{M_s\alpha}{R}\right) - \frac{\cot\phi}{R^2}M_s\,\alpha\end{aligned} \tag{2}$$

The relationship between the stress and moment resultants and generalized displacements, u, w, and α can be defined through the elements of the usual A-B-D stiffness matrices.

The three nonlinear equations of equilibrium and the five stress and moment resultants-displacement relations, along with the corresponding six boundary conditions, are linearized using the Taylor series approach.[2] These sets of linearized governing equations at each load step are then solved iteratively using a Chebyshev-Galerkin spectral method. The de-

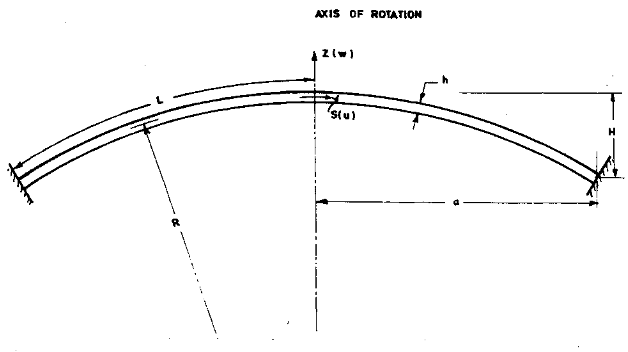

Fig. 1 Geometry and coordinate system for the shallow spherical cap.

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Table 1 Critical pressures of clamped shells

Sl. no.	Material	Critical pressure, P_{cr}: Deep shell theory	Shallow shell theory[1]	Diff., %	w_{max}/h: Deep shell theory	Shallow shell theory[1]	Diff., %
1	Graphite-epoxy $E_L = 16E_T$, $G_{TT} = 0.22E_T$ $\nu_{LT} = 0.3$	15.10	15.0	0.6	0.50	0.60	16.6
2	Boron-epoxy $E_L = 10E_T$, $G_{TT} = 0.33E_T$ $\nu_{LT} = 0.22$	11.10	10.5	5.7	0.61	0.80	23.7
3	Glass-epoxy $E_L = 3E_T$, $G_{TT} = 0.50E_T$ $\nu_{LT} = 0.25$	3.95	3.9	1.3	0.865	1.00	13.5

$P_{cr} = \dfrac{q_{cr}\, a^4}{E_T h^2 H^2}$, $a/h = 10$, $H/a = 0.2$; 5-layered symmetric cross-ply; clamped edges.

Table 2 Effect of H/a on critical pressure

Sl. no.	H/a	Critical pressure, P_{cr}: Deep shell theory	Shallow shell theory[1]	Diff., %	w_{max}/h: Deep shell theory	Shallow shell theory[1]	Diff., %
1	0.1	12.50	—	—	0.82	—	—
2	0.15	11.50	10.80	6.5	0.74	0.96	22.9
3	0.20	12.50	11.85	5.5	0.65	0.90	27.6

$a/h = 15$; 5-layered symmetric cross-ply; clamped; material: boron-epoxy.

Table 3 Effect of stacking sequence on critical pressure

Sl. no.	Stacking	Critical pressure, P_{cr}: Glass-epoxy	Graphite-epoxy
1	(0°/90°)	3.3	5.87
2	$(0°/90°)_2$	4.05	14.50
3	$(0°/90°)_3$	4.50	16.00
4	$(0°/90°)_4$	4.50	15.41

$a/h = 10$; $H/a = 0.2$; clamped.

Table 4 Effect of a/h on critical pressure

Sl. no.	Material	Critical pressure, P_{cr}: $a/h = 10$	$a/h = 20$	$a/h = 50$
1	Graphite-epoxy	5.87	7.50	11.05
2	Glass-epoxy	3.30	3.78	6.00

Lamination 0°/90°, $H/a = 0.2$; clamped.

tails of the incremental solution procedure can be found in Ref. 3. The critical pressure is identified as the applied pressure at which the iterative solution for W_{ave} does not converge within a load step even after a specified maximum number of iterations. The transverse average deflection is defined as

$$W_{ave} = \frac{2R}{L}\int_0^1 \bar{w}\xi \, d\xi \text{ where } \bar{w} = w/h \text{ and } \xi = \frac{s}{L} \tag{3}$$

A convergence tolerance of 0.1% has been adopted in all of the numerical work presented herein.

Numerical Results

In the formulation of a first-order stress deformation theory for laminated shells, a stress correction factor k^2 is conventionally used to account for nonlinear distribution of shear strains through the thickness of the shell. The value of shear correction depends on the sequence, material properties, and relative thickness of each layer. In all of the cases presented herein, the shear correction factors calculated by employing Whitney's method[4] have been used.

Table 1 presents the comparison between the estimates of critical pressure obtained by the present method to those obtained by Xu. It can be observed that, for all of the shells, the critical pressure estimates of the present solution agree well with those of the shallow shell solution. However, there is a large discrepancy with respect to maximum deflections, the shallow shell theory grossly overestimating the deflections.

Table 2 presents a comparison between the results obtained using the present solution with those of the shallow shell solution. It is interesting to note that, at $H/a = 0.1$, whereas the present solution estimates snap-through at $P_{cr} = 12.50$, the shallow shell theory does not predict any snapping at all. For the other two ratios, again there is a fairly good agreement between the estimates of snap-pressure, but the peak deflections are again grossly overestimated by shallow shell theory.

The estimates of the critical pressure and the associated maximum deflections for increasing numbers of layers antisymmetrically stacked are shown in Table 3. It can be observed that the critical pressure increases with increasing number of layers initially but does not change much beyond six layers.

The variations of critical buckling pressures with base-radius to thickness ratio for two layered shells are presented for two different materials in Table 4. It is found that the buckling load increases with this ratio.

Conclusions

The problem of axisymmetric large deflection behavior of antisymmetrically laminated spherical shells is solved using a Chebyshev-Galerkin spectral method. The numerical results presented indicate that, though the critical pressure estimates of the shallow shell theory are reasonably good, for the shell configurations considered herein the associated deflections are overestimated by the shallow shell theory. There is an increase in the buckling pressure of antisymmetrically laminated shells as the number of layers increases for the same total thickness of the shell. The buckling pressure of the laminated shells increases with increasing base-radius to thickness ratio.

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